JOURNAL OF THERMOPHYSICS AND HEAT TRANSFER
Vol. 6, No. 2, April–June 1992

Heat Transfer in the Entrance Region of Semicircular Ducts with Internal Fins

H. Y. Zhang* and M. A. Ebadian†
Florida International University, Miami, Florida 33199

This paper investigates numerically the fluid flow and heat transfer of steady laminar forced convection in the entrance region of semicircular ducts with longitudinal internal fins subjected to a constant wall temperature. The hydrodynamically fully developed flow and developing temperature are solved in this paper for nine geometries. The developing temperature field in the semicircular duct with longitudinal fins is obtained analytically/numerically by solving the energy equation employing the method of line (MOL). The energy equation is reformulated into a system of first-order differential equations controlling the temperature along each line. The representative curves illustrating the isotherms, variations of the bulk temperature and Nusselt number in the entire thermal region of the duct comprised of 3, 7, and 11 fins, and the fin height to duct radius ratio of 0.3, 0.6, and 1.0 are presented graphically. In addition, the number of lines used in the computational domain, the estimate of the thermal entrance length, and the fully developed Nusselt numbers are shown in tabular form for the above-mentioned fin parameters.

Nomenclature

A	=	cross-sectional area, m^2
C	=	factor defined by Eq. (12)
C_p	=	specific heat, J kg^{-1} K^{-1}
D	=	diameter of semicircular duct $D = 2r_i$
D_h	=	hydraulic diameter of duct, m
f	=	fanning friction factor
H	=	relative fin height of $H = h/r_i$
h	=	fin height, m
k	=	thermal conductivity of fluid, W m^{-1} K^{-1}
L	=	characteristic length of duct, m, $L = r_i$
$\dot{m}$	=	mass flow rate of fluid, kg s^{-1}
N	=	number of fins
Nu_T	=	fully developed Nusselt number, hD_h/k
$Nu_{x,T}$	=	local Nusselt number, Eq. (18)
P	=	perimeter, m
Pe	=	Peclet number, $Pe = RePr$
Pr	=	Prandtl number, $Pr = \nu/\alpha$
p	=	pressure, kN m^{-2}
Q_T	=	total heat transfer, W, Eq. (17)
R	=	dimensionless variable, $R = r/r_i$
Re	=	Reynolds number, $\bar{u}D_h/\nu$
r	=	radial coordinate
r_i	=	radius, *m*
T	=	temperature, K
T_b	=	bulk temperature, K
T_e	=	entrance temperature, K
T_w	=	duct wall temperature, K
u, U	=	dimensional and dimensionless velocity of fluid, $U = u/\bar{u}$
u^*	=	dimensionless variable defined by Eq. (2)
$\bar{u}$, $\bar{U}$	=	dimensional and dimensionless mean velocity of fluid
X, x	=	dimensionless and dimensional axial coordinates, $X = x/D_hPe$
α	=	thermal diffusivity of fluid, m^2 s^{-1}
Δ	=	coefficient, Eq. (19)
η	=	generalized independent variable
θ	=	angular coordinate
μ	=	dynamic viscosity of fluid, kg m^{-1} s^{-1}
ν	=	kinematic viscosity of fluid m^{-2} s^{-1}
ρ	=	density of fluid, kg m^{-3}
ϕ_b	=	dimensionless bulk temperature
ω	=	generalized dependent variable

Received May 7, 1990; revision received June 17, 1991; accepted for publication June 19, 1991.

*Graduate Student, Department of Mechanical Engineering. Member AIAA.

†Professor and Chairperson, Department of Mechanical Engineering.

Introduction

LONGITUDINAL fins within ducts are widely used in compact heat exchanger applications as documented by Kays and London[1] and Fraas and Özisik.[2] For many years, the use of this type of tube has been considered in the design of gas-cooled nuclear reactors. A review of many enhancement techniques is presented by Bergles.[3] Inspection of the literature by Eckert et al,[4–6] Kakac et al.,[7] Shah and London,[8] Soloukhin and Martynenko,[9] and Martynenko[10] indicates that significant attention has been devoted in recent years to the investigation of laminar fluid flow and heat transfer in internally finned tubes. However, these investigations are predominantly concerned with forced convection in the developing and fully developed region of the circular duct. Among this research, Hu and Chang,[11] Nandakumar and Masliyah,[12] and Soliman and Feingold[13] studied fully developed velocity distribution and the friction factor. The temperature distribution and the Nusselt number for the condition of uniform heat input axially, with uniform wall temperature circumferentially, is reported by Masliyah and Nandakumar[14] and Soliman and Feingold.[15] In addition, the study of uniform wall temperature, axially and circumferentially, is reported in the literature by Soliman et al.[16] The combined effects of free and forced convection in vertical tubes with radial internal fins have been studied by Prakash and Patankar.[17] They used the finite difference technique to solve the governing equations for velocity and temperature fields. Prakash and Liu[18] investigated laminar flow and heat transfer in the entrance region of internally finned circular ducts. They solved the problem numerically using simultaneous development of velocities and temperature fields. Rustum and Soliman[19] numerically studied laminar forced convection in the entrance region of a tube with longitudinal internal fins. Theoretically, they studied forced convection heat transfer in the developing region of internally finned tubes with fully developed hydrodynamics. Rustum and Soliman[20] also studied experimentally laminar mixed convection in tubes with longitudinal internal fins. They obtained the pressure drop and heat transfer characteristics for laminar flow in a smooth tube and of four tubes with internal longitudinal fins. Finally, an analysis of turbulent flow and heat transfer in internally finned tubes and annuli was reported by Patankar et al.[21] They solved the problem via a mixing length model for fully developed turbulent flow and heat transfer characteristics in tubes and annuli with longitudinal internal fins.

Recently, Manglik and Bergles,[22] Zhang et al.,[23] and Trupp and Liu[24] studied convection heat transfer in the semicircular duct subjected to a uniform wall temperature. Manglik and Bergles[22] obtained the mean bulk temperature and the mean Nusselt number in the entrance region of the semicircular duct using the standard finite difference method, whereas the Zhang et al.[23] solution uses the method of lines (MOL). Trupp and Liu[24] obtained solution to the fully developed laminar heat transfer in circular ducts with isothermal walls. They used the finite difference procedure to obtain the temperature distribution. Unfortunately, these investigations, and others mentioned earlier, do not take into account the effect of longitudinal internal fins. The numerical and experimental investigation of circular ducts with internal fins by Rustum and Soliman[19,20] has clearly demonstrated that heat transfer is enhanced by attaching internal fins.

Therefore, the main focus of the present investigation is to analyze forced convective heat transfer in the entrance region of semicircular ducts with internal fins subjected to a uniform wall temperature (Fig. 1). Additionally, the flow is considered to be laminar, and the physical properties are assumed to be constant. The developing temperature in this geometry with 3, 7, and 11 longitudinal fins is determined by solving the three-dimensional energy equation via the method of lines (MOL),[25] (Fig. 2). According to this method, the transversal derivatives in the energy equation are replaced by a finite-difference formulation, while the axial derivatives remain continuous. Correspondingly, the region of integration is divided into a collection of lines parallel to the axial coordinate. It is widely known that the retention of equal transversal intervals in the presence of irregular boundaries constitutes a complicated feature requiring special equations for the node in its neighborhood.[26] To prevent this complication and difficulty, the two-dimensional grid in the duct cross section is constructed in such a manner that the dividing lines coincide with the regular boundaries, resulting in equal or unequal transversal intervals. From a conceptual point of view, the partial differential energy equation is replaced by a system of ordinary differential equations of the first order, where the independent variable is the axial coordinate. The resulting linear system, accounting for externally coarse grids in conjunction with the appropriate boundary condition, constitutes an initial value problem. Its solution, providing the temperature field, may be readily obtained either analytically, using an eigenvalue subroutine, or numerically, using a Runge-Kutta subroutine.[27]

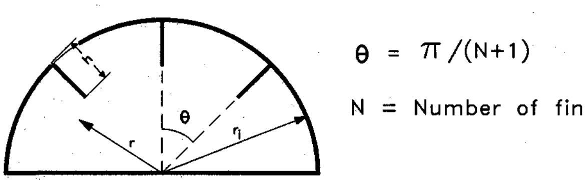

Fig. 1 Coordinate system for the semicircular duct.

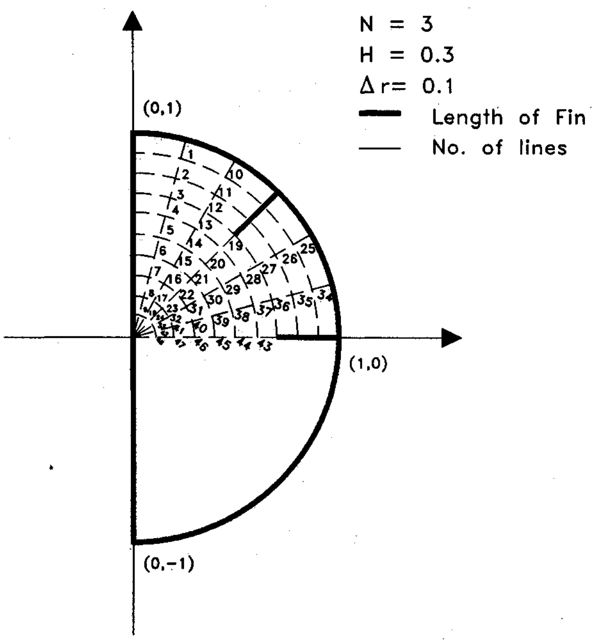

Fig. 2 Coordinate system for the coarse grid with H = 0.3 and N = 0.3

Analytical Formulation

The problem considered here is that of a semicircular tube, as shown in Fig. 1. The semicircular duct has a variable number of straight internal fins N which are evenly distributed around the inner circumference of the duct with a different relative fin height H. The fins are assumed to be of zero thickness. The flow is assumed to be fully developed hydrodynamically, but thermally developing in the entrance region of the semicircular duct. The flow enters the duct with velocity $\bar{u}$ and a uniform temperature T_e. The conservation of momentum equation is expressed as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = \frac{1}{\mu}\frac{\mathrm{d}p}{\mathrm{d}x} \tag{1}$$

where x, r, θ are the axial, radial, and angular coordinates of the duct, respectively. Additionally, μ is the dynamic viscosity, and $\mathrm{d}p/\mathrm{d}x$ is the pressure gradient in the axial coordinate. Upon introduction of the dimensionless variable

$$u^* = \left\{\frac{\mu u}{r_i^2(-\mathrm{d}p/\mathrm{d}x)}\right\}, \qquad R = \frac{r}{r_i} \tag{2}$$

Equation (1) with reference to Fig. 2 is transformed into

$$\frac{\partial^2 u}{\partial R^2} + \frac{1}{R}\frac{\partial u^*}{\partial R} + \frac{1}{R^2}\frac{\partial^2 u^*}{\partial \theta^2} + 1 = 0 \tag{3}$$

The preceding equation, accounting for the characteristic length, is subjected to the following boundary conditions:

$$\begin{aligned} u^* = 0 \quad @ \quad & R = 1 & & 0 \le \theta \le \pi \\ & (1 - H) \le R \le 1, & & \theta = \frac{\pi}{(N+1)} \\ & 0 \le R \le 1, & & \theta = 0,\ \theta = \pi \end{aligned} \tag{4}$$

and

$$\frac{\partial u^*}{\partial \theta} = 0 \quad @ \quad 0 < R < (1 - H), \quad \theta = \frac{\pi}{2} \tag{5}$$

From the definition of fRe[8]

$$fRe = \frac{D_h^2}{2\mu\bar{u}}\left[-\frac{\mathrm{d}p}{\mathrm{d}x}\right] \tag{6}$$

with

$$D_h = 2R_i \tag{6a}$$

$$D_h = \frac{2\pi R}{\pi + 2} \tag{6b}$$

$$D_h = \frac{\pi D^2}{(\pi + 2)D + 4Nh} \tag{6c}$$

where Eqs. (6a), (6b), and (6c) are defined based on the diameter of a circular pipe, the hydraulic diameter of a bare semicircular pipe, and the hydraulic diameter of a semicircular duct with fins, respectively. It can easily be seen that

$$U = 2\frac{r_i^2}{D_h^2}(fRe)\, u^* \tag{7}$$

where u^* is taken from the solution of the conservation of momentum of Eq. (3).

Energy Equation

The conservation of energy equation for the thermal developing region is given by

$$\rho C_p u \frac{\partial T}{\partial x} = k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right] \tag{8}$$

Likewise, upon introduction of the following set of dimensionless variables

$$U = \frac{u}{\bar{u}}, \qquad \phi = \frac{T - T_w}{T_e - T_w}$$

$$X = \frac{x}{D_h Pe}, \qquad Re = \frac{\bar{u}D_h}{\nu}, \qquad Pe = RePr \tag{9}$$

Equation (8) may be rewritten as

$$U\frac{r_i^2}{D_h^2}\frac{\partial \phi}{\partial X} = \frac{\partial^2 \phi}{\partial R^2} + \frac{1}{R}\frac{\partial \phi}{\partial R} + \frac{1}{R^2}\frac{\partial^2 \phi}{\partial \theta^2} \tag{10}$$

Furthermore, the above equation can be defined in terms of the fanning friction factor by introducing Eq. (7) into Eq. (10). The outcome of the transformation is written as

$$\frac{\partial \phi}{\partial X} = C\left(\frac{\partial^2 \phi}{\partial R^2} + \frac{1}{R}\frac{\partial \phi}{\partial R} + \frac{1}{R^2}\frac{\partial^2 \phi}{\partial \theta^2}\right) \tag{11}$$

where

$$C = \frac{D_h^4}{2r_i^4}\frac{1}{u^*(fRe)} \tag{12}$$

The applicable boundary conditions for Eq. (11) are

$$\phi = 0 \quad @ \quad R = 1 \qquad 0 \le \theta \le \pi$$

$$(1 - H) \le R \le 1, \quad \theta = \frac{\pi}{(N + 1)}$$

$$0 \le R \le 1, \qquad \theta = 0,\ \theta = \pi \tag{13}$$

and

$$\frac{\partial \phi}{\partial \theta} = 0 \quad @ \quad 0 < R < (1 - H), \quad \theta = \frac{\pi}{2}$$

$$\phi = 1 \quad @ \quad X = 0 \tag{14}$$

Thermal Quantities of Interest

Momentum Equation

The hydrodynamic characteristics of the fluid flow in a duct of arbitrary cross section are usually described by the mean velocity and the friction factor.

The dimensionless mean velocity, $\bar{U}$, is obtained from integrating the equation

$$\bar{U} = \left[\int_A U\, \mathrm{d}A\right]\Big/ A \tag{15}$$

From this, fRe may now be calculated.

Energy Equation

The thermal characteristics of fluid flow inside any duct may be represented by a dimensionless bulk temperature as

$$\phi_b = \int_A U\phi\, \mathrm{d}A \Big/ \int_A U\, \mathrm{d}A \tag{16}$$

From this, the total heat transfer Q_T between the inlet, $x = 0$, and a certain duct length L may be easily calculated via an overall energy balance, wherein

$$Q_T = \dot{m}C_p[T_e - T_b(L)] \tag{17}$$

The total heat transfer Q_T may also be computed from the local Nusselt number

$$Nu_{x,T} = -\frac{1}{4}\left(\frac{D_h^2}{r_i^2}\right)\frac{1}{\phi_b}\frac{\mathrm{d}\phi_b}{\mathrm{d}X} \tag{18}$$

Numerical Solution Procedure

The velocity profile, $u^*(R, \theta)$, has been computed by the finite difference method using the grid shown in Fig. 2. In general, the corresponding central formulation for the second derivative accounting for nonuniform intervals is given by

$$\left.\frac{\partial^2 \omega}{\partial \eta^2}\right|_i = \frac{\omega_{i+1} - (\Delta + 1)\omega_i + \omega_{i-1}}{(\Delta + 1)\Delta\eta^+\Delta\eta^-/2} \tag{19}$$

where ω is an arbitrary variable and $\Delta = \Delta\eta^+/\Delta\eta^-$.

The temperature distribution, $\phi(R, \theta, X)$, has been determined by the hybrid method of lines (MOL). The method of lines has been described by Liskovets[25] as a solution technique that transforms a partial differential equation into an appropriate system of ordinary differential equations. For a parabolic PDE involving three independent variables, as in Eq. (10), the region of integration may be divided into straight lines parallel to the axial coordinate X. Accordingly, the axial derivative $\partial\phi/\partial X$ remains continuous while the radial and angular derivatives $\partial^2\phi/\partial R^2$ and $\partial^2\phi/\partial\theta^2$ are represented by finite difference formulations of unknown quantities on the same line and neighboring lines. Hence, this simple mathematical concept generates a system of ordinary differential equations of the first order, where the dependent variables are described along each line in terms of a single independent variable X.

In light of the foregoing, the differential difference equation applicable to each line (i, j) in a semicircular duct may be written as follows:

$$\frac{\mathrm{d}\phi_{ij}}{\mathrm{d}X} = C\left(\frac{\phi_{i+1,j} - (\Delta + 1)\phi_{ij} + \phi_{i-1,j}}{(\Delta + 1)\Delta R^+\Delta R^-/2} + \frac{1}{R}\frac{\phi_{i+1,j} + (\Delta^2 - 1)\phi_{i,j} - \Delta^2\theta_{i-1,j}}{(\Delta + 1)\Delta R^+} + \frac{1}{R^2}\frac{\phi_{i,j-1} - (\Delta + 1)\phi_{ij} + \phi_{i,j-1}}{(\Delta+1)\Delta\theta^+\Delta\theta^-/2}\right) \tag{20}$$

Accordingly, the entrance boundary condition in the integration domain may be rewritten in discrete form as

$$\phi_{i,j}(0) = 1 \qquad i = 0, 1, \ldots I - 1$$
$$j = 0.1, \ldots J - 1 \qquad (21)$$

Mathematically speaking, Eq. (21) constitutes an initial value problem, which may be solved either a) analytically, employing a eigenvalue subroutine,[27] or b) numerically, utilizing a Runge-Kutta algorithm. In this paper, the numerical technique utilizing the Runge-Kutta algorithm with a step size of $\Delta X = 10^{-6}$ is used. The main advantage of the former integration procedure is that it provides closed-form expressions for the temperature field, $\phi(X)$. Likewise, closed-form expressions for distribution of the mean-bulk temperature, ϕ_b, and the local Nusselt number, $Nu_{x,T}$, may be readily determined.

Results and Discussion

The dimensionless mean velocity, $\bar{U}$, Eq. (15), the mean bulk temperature, ϕ_b, Eq. (16), and the local Nusselt numbers, $Nu_{x,T}$, Eq. (18), with longitudinal fins has been computed numerically in the development region of the semicircular duct, using the method of lines (MOL). The semicircular duct is considered to be smooth, with longitudinal internal fins. The results are presented for nine different geometries. These include a combination of fins, N = 3, 7, or 11, with a relative fin height of H = 0.3, 0.6, 1.0, respectively. In particular, the lines of isotherms for two different numbers of fins, the bulk temperature and the local Nusselt number have been illustrated graphically in Figs. 5 through 10. As can be seen in Fig. 2, computation of the velocity and the temperature distribution is based on 48 lines for H = 0.3. Equally spaced grids are used in both radial and axial directions. To ensure the accuracy of the numerical solution, an independent grid size test has also been conducted in this study. Tables 1 and 2 depict the results of this test with different grid sizes based on Eqs. (6b) and (6c). The number of fins and relative fin heights used in this test case are N = 3, 7, 11 and H = 0.3, 0.6, 1.0, respectively. Inspection of the data indicates that very close results for the Nusselt number can be obtained with the number of lines specified in this table. Therefore, the smaller grid size is mainly used throughout this study for

Table 1 The effects of grid size on the fully developed Nusselt number based on Eq. (6b)

Relative fin height	Number of fins 3		7		11	
	Lines	Nu_T	Lines	Nu_T	Lines	Nu_T
0.3	84	6.593	96	7.640	90	7.738
	174	6.350	192	7.425	198	7.543
0.6	78	13.955	120	26.478	126	23.035
	162	13.613	240	25.983	252	22.524
1.0	63	12.400	54	30.156	54	53.760
	324	12.399	324	29.983	324	53.492

Table 2 The effects of grid size on the fully developed Nusselt number based on Eq. (6c)

Relative fin height	Number of fins 3		7		11	
	Lines	$Nu_{x,T}$	Lines	$Nu_{x,T}$	Lines	Nu_T
0.3	84	3.618	96	2.315	90	1.484
	174	3.485	192	2.250	198	1.447
0.6	78	4.829	120	3.818	126	1.811
	162	4.711	240	3.747	252	1.770
1.0	63	2.652	54	2.175	54	1.929
	324	2.640	324	2.163	324	1.920

Table 3 Estimate of thermal entrance length

Relative fin height	Number of fins 3	7	11
	X	X	X
0.3	0.2515	0.18000	0.04400
0.6	0.1175	0.04750	0.01600
1.0	0.0150	0.00757	0.00405

Table 4 Comparison of fully developed local Nusselt number for relative fin height H = 1.0

Diameter	Number of fins	Number of lines	Present study	Prakash and Liu[18]	Trupp and Lau[24]
Eq. (6a)	3	63	33.375		
		324	33.210	33.250	—
	7	54	80.772		
		324	80.306	80.550	—
	11	54	143.997		
		324	143.279	143.700	—
Eq. (6b)	3	63	12.460		
		324	12.398	—	—
	7	54	30.155		
		324	29.981	—	—
	11	54	53.760		
		324	53.492	—	—
Eq. (6c)	3	63	2.652		
		324	2.640	2.643	2.613
	7	54	2.175		
		324	2.163	2.170	2.132
	11	54	1.929		
		324	1.919	1.925	1.890

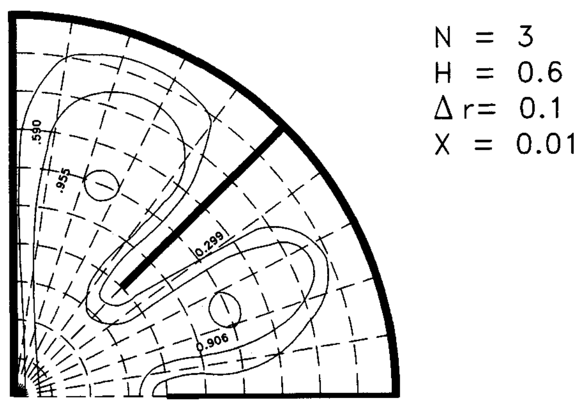

Fig. 3 Lines of isotherms for N = 3, H = 0.6, and X = 0.01.

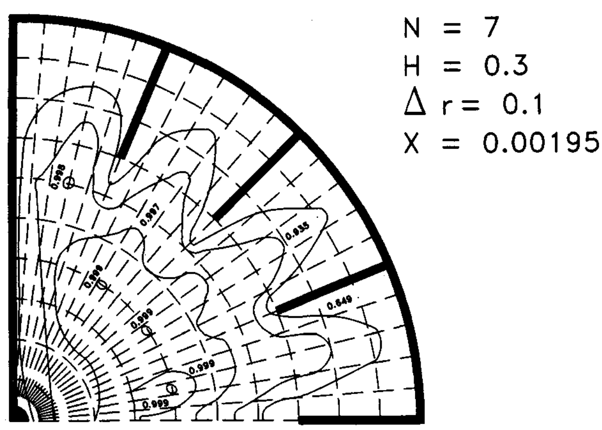

Fig. 4 Lines of isotherms for N = 7, H = 0.3, and X = 0.00195.

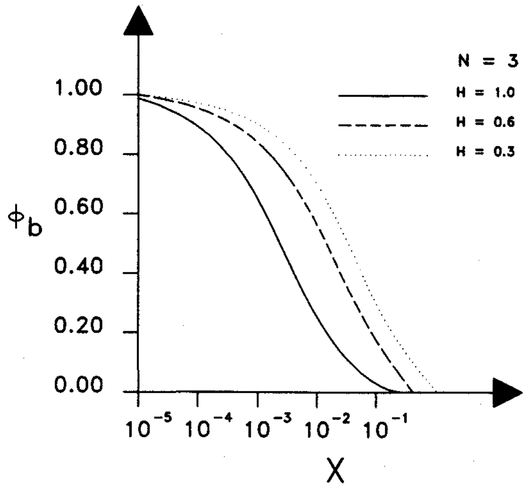

Fig. 5 Variation of the bulk temperature for $N = 3$.

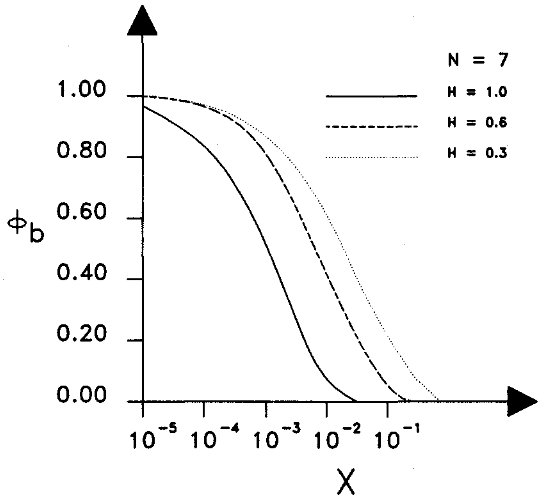

Fig. 6 Variation of the bulk temperature for $N = 7$.

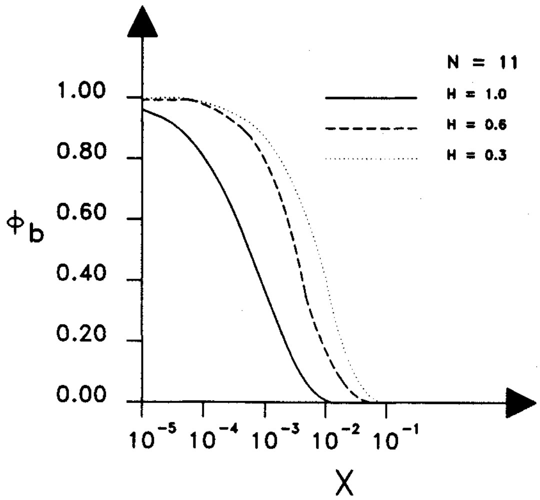

Fig. 7 Variation of the bulk temperature for $N = 11$.

computation of the Nusselt number. Accuracy of the thermal results for the semicircular duct with longitudinal fins may be improved by adding more lines. As mentioned earlier, Tables 1 and 2 compare the grids for the nine different configurations of fins. The thermal entrance length for the nine different geometries are listed in Table 3. Accordingly, this length as defined by Shah and London[8] is the length required for the local Nusselt number to equal 1.05 times its fully developed value. The results in Table 3 are based on this definition.

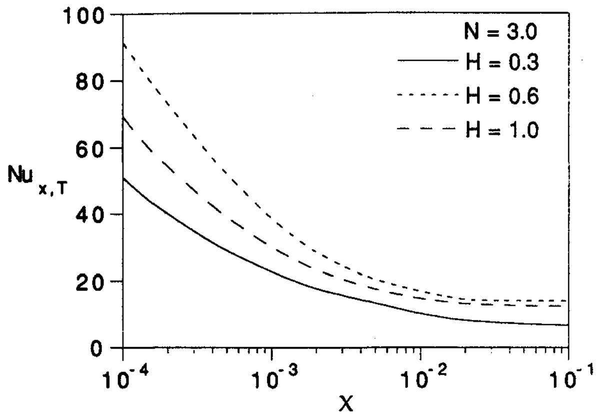

Fig. 8 Variation of the local Nusselt number for $N = 3$.

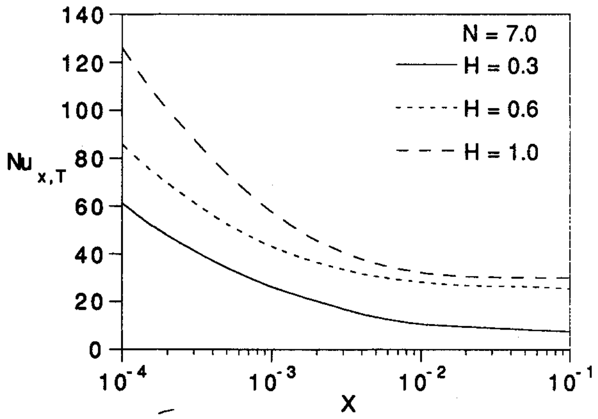

Fig. 9 Variation of the local Nusselt number for $N = 7$.

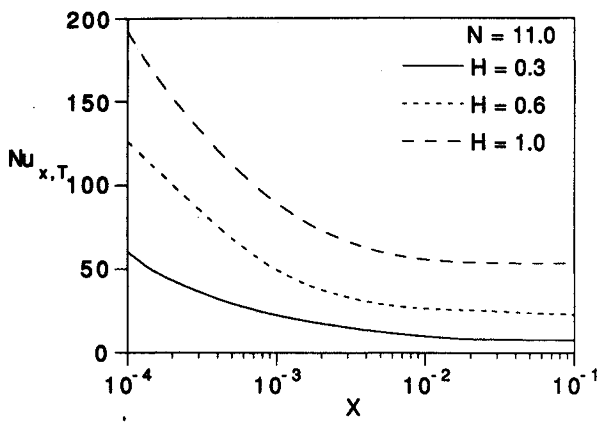

Fig. 10 Variation of the local Nusselt number for $N = 11$.

Table 4 presents a comparison of the fully developed local Nusselt number values for semicircular ducts with the circular ducts of Prakash and Liu[18] and the circular duct of Trupp and Lau[24] for different values of N and a relative fin height of $H = 1.0$. It is seen from this table, and also from Fig. 2, that three fins for the semicircular duct are equivalent to eight fins for the circular duct for a relative fin height of $H = 1.0$. Accordingly, seven and eleven fins for the semicircular duct correspond to sixteen and twenty-four fins for the circular duct, respectively, in the same manner. It is further evident from this table that increasing both the number of fins N and the relative fin height H for the semicircular duct will enhance the Nusselt number dramatically.

Inspection of this table reveals that the results for the fully developed local Nusselt number is based on three different hydraulic diameters. The first section of this table provides a comparison of the local Nusselt number for the semicircular duct with the results of Prakash and Liu[18] based on the duct diameter, Eq. (6a). Further inspection indicates an excellent agreement between the results of the fully developed Nusselt number. The second part of this table provides these results based on Eq. (6b). Finally, the last part of the table provides a comparison of the results of the fully developed local Nusselt number for the semicircular duct with those given by Prakash and Liu[18] and Trupp and Lau[24] based on Eq. (6c). Further investigation of these data indicated an excellent agreement between the results presented.

Loops and isotherms for the two cases of $N = 3$ with $H = 0.6$, and $N = 7$ with $H = 0.3$ are shown in Figs. 3 and 4, respectively. Figures 3 and 4 illustrate that the lowest temperature takes place along the semicircular duct with two and four eyes, respectively, whereas the highest temperature occurs at the eye of each loop. Therefore, it can be concluded that increasing value of N and H will increase the number of eyes in each loop and also eye points in each fin.

Figures 5 through 7 represent variation of the bulk temperature against the thermal entrance length, $X(10^{-5} < X < 10^{-1})$, for $N = 3$, 7, and 11. Also, in each figure, the effect of the relative fin height is illustrated by showing results for $H = 0.3$, 0.6, and 1.0. It is evident from these figures that increasing H delays the development of the bulk temperature.

Figures 8 through 10 show variation of the local Nusselt number against the entrance length X for the same combination of bulk temperatures. As expected, the Nusselt number is larger near the inlet and decreases asymptotically to the fully developed flow. Comparison of the present results for the mean Nusselt number with Manglik and Bergles[22] and Zhang et al.[23] indicates an excellent agreement for the case of the base semicircular duct.

Concluding Remarks

A simple computational procedure, the method of lines (MOL), based on standard finite-difference analogs, has been adopted for the analysis of laminar forced convection in the thermal entrance region of the semicircular duct with longitudinal fins. The nodes have been placed carefully to avoid the usual difficulties associated with the irregular boundary. The use of coarse grids in the cross section of the duct yields a relatively small system of first order ordinary differential equations. Hence, this system can be readily solved by either analytical or numerical techniques. It is further concluded that by increasing the number of fins and relative fin height, the Nusselt number increases accordingly, and thus delays the development of the local Nusselt number in the entrance region of the duct.

Acknowledgment

The result presented in this paper were obtained in the course of research sponsored by The National Science Foundation under Grant No. CTS-9017732.

References

[1]Kays, W. M., and London, A. L., *Compact Head Exchangers,* 2nd ed., McGraw-Hill, New York, 1964.

[2]Fraas, A. P., and Ozisik, M. N., *Heat Exchanger Design,* 2nd ed., Wiley, New York, 1965.

[3]Bergles, A. E., "Survey and Evaluation of Techniques to Augment Convective Heat and Mass Transfer," *Progress in Heat and Mass Transfer,* 1, Pergamon Press, Elmsford, New York, 1969, pp. 331–424.

[4]Eckert, E. R. G., et al., "Heat Transfer—A Review of the 1985 Literature," *International Journal of Heat Mass Transfer,* Vol. 29, No. 12, 1986, pp. 1767–1842.

[5]Eckert, E. R. G., et al., "Heat Transfer—A Review of the 1986 Literature," *International Journal of Heat Mass Transfer,* Vol. 30, No. 12, 1987, pp. 2449–2523.

[6]Eckert, E. R G., et al., "Heat Transfer—A Review of the 1987 Literature," *International Journal of Heat Mass Transfer,* Vol. 31, No. 12, 1988, pp. 2401–2488.

[7]Kakac, S., Shah, R. K., and Aung, W., *Handbook of Single-Phase Convection Heat Transfer,* Wiley Interscience, New York, 1987.

[8]Shah, R. K., and London, A. L., *Laminar Flow Forced Convection in Ducts,* Academic Press, New York, 1978.

[9]Soloukhin, R. I., and Martynenko, O. G., "Heat and Mass Transfer Bibliography—Soviet Literature," *International of Journal Heat Mass Transfer,* Vol. 26, No. 12, 1983, pp. 1771–1781.

[10]Martynenko, O. G., "Heat and Mass Transfer Bibliography—Soviet Literature," *International Journal of Heat Mass Transfer,* Vol. 31, No. 12, 1988, pp. 2489–2503.

[11]Hu, M. H., and Chang, Y. P., "Optimization of Finned Tubes for Heat Transfer in Laminar Flow," *Journal of Heat Transfer,* No. 3, 1973, pp. 332–338.

[12]Nandakumar, K., and Masliyah, J. H., "Fully Developed Viscous Flow in Internally Finned Tubes," *The Chemical Engineering Journal,* Vol. 10, 1975, pp. 113–120.

[13]Soliman, H. M., and Feingold, A., "Analysis of Fully Developed Laminar Flow in Internally Finned Tubes," *The Chemical Engineering Journal,* Vol. 14, 1977, pp. 119–128.

[14]Masliyah, J. H., and Nandakumar, K., "Heat Transfer in Internally Finned Tubes," *Journal of Heat Transfer,* Vol. 98, No. 2, 1976, pp. 257–261.

[15]Soliman, H. M., and Feingold, A., "Analysis of Heat Transfer in Internally Finned Tubes under Laminar Flow Condition," *Proceedings of the Sixth International Heat Transfer Conference,* Vol. 2, 1978, pp. 571–576.

[16]Soliman, H. M., Chau, T. S., and Trupp, A. C., "Analysis of Laminar Heat Transfer in Internally Finned Tubes with Uniform Outside Wall Temperature," *Journal of Heat Transfer,* Vol. 102, No. 3, pp. 598–604.

[17]Prakash, C., and Patankar, S. V., "Combined Free and Forced Convection in Vertical Tubes with Radial Internal Fins," *Journal of Heat Transfer,* Vol. 103, No. 3, 1981, pp. 566–572.

[18]Prakash, C., and Liu, Y. D., "Analysis of Laminar Flow and Heat Transfer in the Entrance Region of Internally Finned Circular Ducts," *Journal of Heat Transfer,* Vol. 107, 1985, pp. 84–91.

[19]Rustum, I. M., and Soliman, H. M., "Numerical Analysis of Laminar Forced Convection in the Entrance Region of Tubes with Longitudinal Internal Fins," *Journal of Heat Transfer,* Vol. 110, No. 2, 1988, pp. 310–313.

[20]Rustum, I. M., and Soliman, H. M., "Experimental Investigation of Laminar Mixed Convection in Tubes with Longitudinal Internal Fins," *Journal of Heat Transfer,* Vol. 110, No. 2, 1988, pp. 366–372.

[21]Patanker, S. V., Ivanovic, M., and Sparrow, E. M., "Analysis of Turbulent Flow and Heat Transfer in Internally Finned Tubes and Annuli," *Journal of Heat Transfer,* Vol. 101, No. 1, 1979, pp. 29–37.

[22]Manglik, R. M., and Bergles, A. E., "Laminar Flow Heat Transfer in a Semi-Circular Tube with Uniform Wall Temperature," *International Journal of Heat and Mass Transfer,* Vol. 31, No. 3, 1988, pp. 625–636.

[23]Zhang, H. Y., Ebadian, M. A., and Campo, A., "Computation of Heat and Fluid Flow in Ducts of Arbitrary Cross Section," *International Journal of Numerical Heat Transfer,* Part A, Vol. 17, No. 2, 1990, pp. 231–248.

[24]Trupp, A. C., and Lau, A. C. Y., "Fully Developed Laminar Heat Transfer in Circular Sector Duct with Isothermal Walls," *Journal of Heat Transfer,* Vol. 106, No. 2, 1984, pp. 467–469.

[25]Liskovets, D. A., "The Method of Lines (Review)," *Differential Equation,* Vol. 1, 1965, pp. 1308–1323.

[26]Carnahan, B., Luther, H. A., and Wilkes, J. O., *Applied Numerical Methods,* John Wiley, New York, 1969.

[27]*IMSL Library Reference Manual,* IMSL, Inc., Houston, TX, 1982.